Physics Letters B 773 (2017) 290–294

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

On the Smarr formula for rotating dyonic black holes

Gérard Clément [a,*], Dmitri Gal'tsov [b,c]

[a] LAPTh, Université Savoie Mont Blanc, CNRS, 9 chemin de Bellevue, BP 110, F-74941 Annecy-le-Vieux cedex, France
[b] Department of Theoretical Physics, Faculty of Physics, Moscow State University, 119899, Moscow, Russia
[c] Kazan Federal University, 420008 Kazan, Russia

ARTICLE INFO

Article history:
Received 11 July 2017
Accepted 17 August 2017
Available online 24 August 2017
Editor: M. Cvetič

ABSTRACT

We revisit the derivation by Tomimatsu of the generalized Komar integrals giving the mass and angular momentum of rotating Einstein–Maxwell black holes. We show that, contrary to Tomimatsu's claim, the usual Smarr formula relating the horizon mass and angular momentum still holds in the presence of both electric and magnetic charges. The simplest case is that of dyonic Kerr–Newman black holes, for which we recover the modified Smarr formula relating the asymptotic mass and angular momentum, the difference between asymptotic and horizon masses being equal to the sum of the two Dirac string masses. Our results apply in particular to the case of dyonic dihole solutions which have been investigated recently.

1. Introduction

The Smarr formula [1] relating the mass, angular momentum, entropy and electric charge of black holes was originally designed for the electrically charged Kerr–Newman solution. Its possible generalizations were extensively analyzed by Carter [2] on the basis of Komar integrals [3]. More general solutions describing axisymmetric configurations of multiple rotating black holes (possibly joined by strings) endowed with electric and also magnetic charges were discussed recently. These solutions usually have a simple description in terms of Ernst potentials, while the metric and the electromagnetic potentials are rather complicated. For such situations Tomimatsu designed his original formulas [4,5] expressing black hole parameters in terms of both the metric variables and the Ernst potentials taken on the symmetry axis. These formulas suggested in 1984 were successfully applied for multiple electrically charged rotating black holes [6].

On the other hand, when the Tomimatsu formulas (completed by an analogous expression for magnetic charge) were applied to multi-dyons [5,7–10], it was observed that the resulting values for the black hole parameters failed to obey the standard Smarr relation, but obeyed a generalized Smarr relation with both electric and magnetic contributions. However, the derivation by Tomimatsu [5] gives little details of the underlying calculations, so to clarify the situation a new derivation is necessary. Here such a derivation is presented, showing that in the original Tomimatsu formulas an important term is missing. Correcting the Tomimatsu formulas, we obtain a new version which, when applied to dyons, leads to the standard Smarr relation between the local horizon mass, angular momentum and electric charge.

In passing we establish the crucial role played by the Dirac strings associated with magnetic monopoles in the mass and angular momentum balance equations. We show that for the Kerr–Newman solution with both electric and magnetic charges the Dirac strings are endowed with non-zero generalized Komar masses which should be taken into account in the Smarr formula for the total mass. We also find that the symmetric choice of gauge for the vector potential (with both North and South pole Dirac strings present with equal weights) for dyons is essential to achieve the total angular momentum balance of the configuration.

2. Generalized Komar mass and angular momentum

We first review the generalized Komar formulas [2] giving the masses and angular momenta of extended sources of Einstein–Maxwell fields. The Komar mass and angular momentum for an asymptotically flat, stationary, axisymmetric configuration are given by the integrals over a spacelike 2-surface at infinity [3][1]:

* Corresponding author.
E-mail addresses: gclement@lapth.cnrs.fr (G. Clément), galtsov@phys.msu.ru (D. Gal'tsov).

[1] We use the metric signature $(-+++)$ and the convention $d\Sigma_{\mu\nu} = 1/2\sqrt{|g|}\,\epsilon_{\mu\nu\lambda\tau}dx^{\lambda}dx^{\tau}$ with $\epsilon_{t\rho z\varphi} = 1$ in Weyl coordinates. We will label t, φ by an index a, and the remaining coordinates ρ, z by i, j. In Sect. 4 we will also use

http://dx.doi.org/10.1016/j.physletb.2017.08.041